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Solution Manual to Mas-Colell

Chapter 7. Basic Elements of Noncooperative Games

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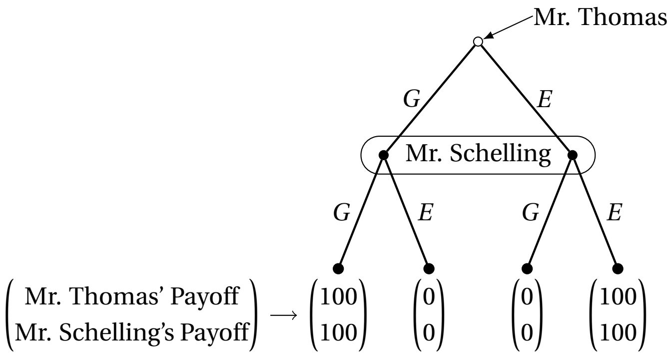
7.C. The Extensive Form Representation of a Game

Exercise 7.C.1.

Denote Grand Central Station by G and denote Empire State Building by E.



7.D. Strategy and the normal Form Representation of a Game

Exercise 7.D.1.

$\Pi_{n=1}^{N} M_n$

Exercise 7.D.2.

		Player 2	
		H	T
Player 1	H	-1,1	1,-1
	T	1,-1	-1,1

7.E. Randomized Choice

Exercise 7.E.1.

(a) There are 3 information set for player 1. Hence, the strategies for player 1 are 3-tuples. The first entry stands for the action in root. The second one is the action in the information set after the M action. The all possible strategies for player 1 are:

$$S_1 = \{(L,x,x),(L,x,y),(L,y,x),(L,y,y),(M,x,x),(M,x,y),$$
$$(M,y,x),(M,y,y),(R,x,x),(R,x,y),(R,y,x),(R,y,y)\}.$$

On the other hand, player 2 has only one information set. The possible strategies for player 2 are:

$$S_2 = \{(l,r)\}.$$

(b) Suppose that player 1, at the root, plays L, M, R with probabilities p_1, p_2, p_3 respectively; at information set after M action , player 1 plays x, y with probabilities of q_1 and q_2 respectively; at information set after R action , player 1 plays x, y with probabilities r_1 and r_2 respectively.

Besides, suppose that player 2 plays l and r with probabilities $\sigma(l)$ and $\sigma(r)$ respectively. Thus, if player 1 uses the behavior strategy above and player 2 uses the mix strategy above, the probability distribution of each terminal node will be:

$$Pr(T_0) = p_1; Pr(T_1) = p_2\sigma(l)q_1; Pr(T_2) = p_2\sigma(l)q_2; Pr(T_3) = p_2\sigma(r)q_1;$$
$$Pr(T_4) = p_2\sigma(r)q_2; Pr(T_5) = p_3\sigma(l)r_1; Pr(T_6) = p_3\sigma(l)r_2;$$
$$Pr(T_7) = p_3\sigma(r)r_1; Pr(T_8) = p_3\sigma(r)r_2$$

Now the following mixed strategy for player 1 is realization equivalent to the above behavior strategy:

$$Pr((L,x,x)) = p_1, Pr((M,x,x)) = p_2q_1,$$
$$Pr((M,y,x)) = p_2q_2, Pr((R,x,x)) = p_3r_1$$
$$Pr((R,x,y)) = p_3r_2$$

(c) Suppose the probability distribution of the strategies is as following:

Action	Probability	Action	Probability
(L,x,x)	p_1	(M,y,x)	p_7
(L,x,y)	p_2	(M,y,y)	p_8
(L,y,x)	p_3	(R,x,x)	p_9
(L,y,y)	p_4	(R,x,y)	p_{10}
(M,x,x)	p_5	(R,y,x)	p_{11}
(M,x,y)	p_6	(R,y,y)	p_{12}

It player 2's strategy is σ, the probability of reaching each terminal node would be:

Node	Probability	Node	Probability
T_0	$p_1+p_2+p_3+p_4$	T_5	$(p_9+p_{10})\sigma(l)$
T_1	$(p_5+p_6)\sigma(l)$	T_6	$(p_{11}+p_{12})\sigma(l)$
T_2	$(p_7+p_8)\sigma(l)$	T_7	$(p_9+p_{10})\sigma(r)$
T_3	$(p_5+p_6)\sigma(r)$	T_8	$(p_{11}+p_{12})\sigma(r)$
T_4	$(p_7+p_8)\sigma(r)$		

Hence, the following behavior strategy for player 1 is realization equivalent:

Information set	Node	Probability
	L	$p_1+p_2+p_3+p_4$
1	M	$p_5+p_6+p_7+p_8$
	R	$p_9+p_{10}+p_{11}+p_{12}$
2	x	$\frac{p_5+p_6}{p_5+p_6+p_7+p_8}$
	y	$\frac{p_7+p_8}{p_5+p_6+p_7+p_8}$
3	x	$\frac{p_9+p_{10}}{p_9+p_{10}+p_{11}+p_{12}}$
	y	$\frac{p_{11}+p_{12}}{p_9+p_{10}+p_{11}+p_{12}}$

(d) In the new information set, player 1 can not distinguish between he choose M or R. Hence, it is not perfect recall. The result of (b) still holds. That is, there exists a mixed strategy for player 1 which is realization equivalent to any behavior strategy. Suppose that player 1, at the root, plays L, M, R with probabilities of p_1, p_2, p_3 respectively; at the merged information set, he plays x and y with probabilities of q_1 and q_2 respectively. If player's mixed strategy is σ, then the probabilities of reaching each terminal nodes will be:

Node	Probability	Node	Probability
T_0	p_1	T_5	$p_3\sigma(l)q_1$
T_1	$p_2\sigma(l)q_1$	T_6	$p_3\sigma(l)q_2$
T_2	$p_2\sigma(l)q_2$	T_7	$p_3\sigma(r)q_1$
T_3	$p_2\sigma(r)q_1$	T_8	$p_3\sigma(l)q_2$
T_4	$p_2\sigma(r)q_2$		

The following mixed strategy for player 1 is realization equivalent:

Strategy	Probability
(L, x)	p_1
(M, x)	$p_2 q_1$
(M, y)	$p_2 q_2$
(R, x)	$p_3 q_1$
(R, y)	$p_3 q_2$

However, there does not always exists behavior strategy that is realization equivalent to a mixed strategy. To provide a counterexample, we assume that player 1 plays (M, x) and (R, y) with equal probability, and Player 2 plays l. Hence, the probabilities reaching terminal node would be

Node	Probability	Node	Probability
T_0	0	T_5	0
T_1	0.5	T_6	0.5
T_2	0	T_7	0
T_3	0	T_8	0
T_4	0		

Then we assume that there is realization equivalent (p_1, p_2, p_3) and (q_1, q_2) defined like above. Base on these probabilities, the probabilities reaching terminal node would be

Node	Probability	Node	Probability
T_0	p_1	T_5	$p_3 q_1$
T_1	$p_2 q_1$	T_6	$p_3 q_2$
T_2	$p_2 q_2$	T_7	0
T_3	0	T_8	0
T_4	0		

Thus, $p_2 q_1 = 0.5 \Rightarrow p_2 \neq 0$. However, $p_2 q_2 = 0$. It implies $q_2 = 0$, but $p_3 q_2 = 0.45$ requires $q_2 \neq 0$, a contradiction.